TDC part II (5) paper III Group A Optics

Dr. MD NAIYAR PERWEZ

LAWS OF REFRACTION USING FERMAT'S PRINCIPLE

Suppose $P_1P_2P_3P_4$ is a horizontal plane lying in Z-X plane which separates two media. μ_1 and μ_2 ($\mu_2 > \mu_1$) are refractive indices of the media above and below the separating plane $P_1P_2P_3P_4$ respectively. A and C are two points above and below the separating plane in vertical plane ABCD. Light coming from the point A is refracted towards the point C as shown in fig-5.

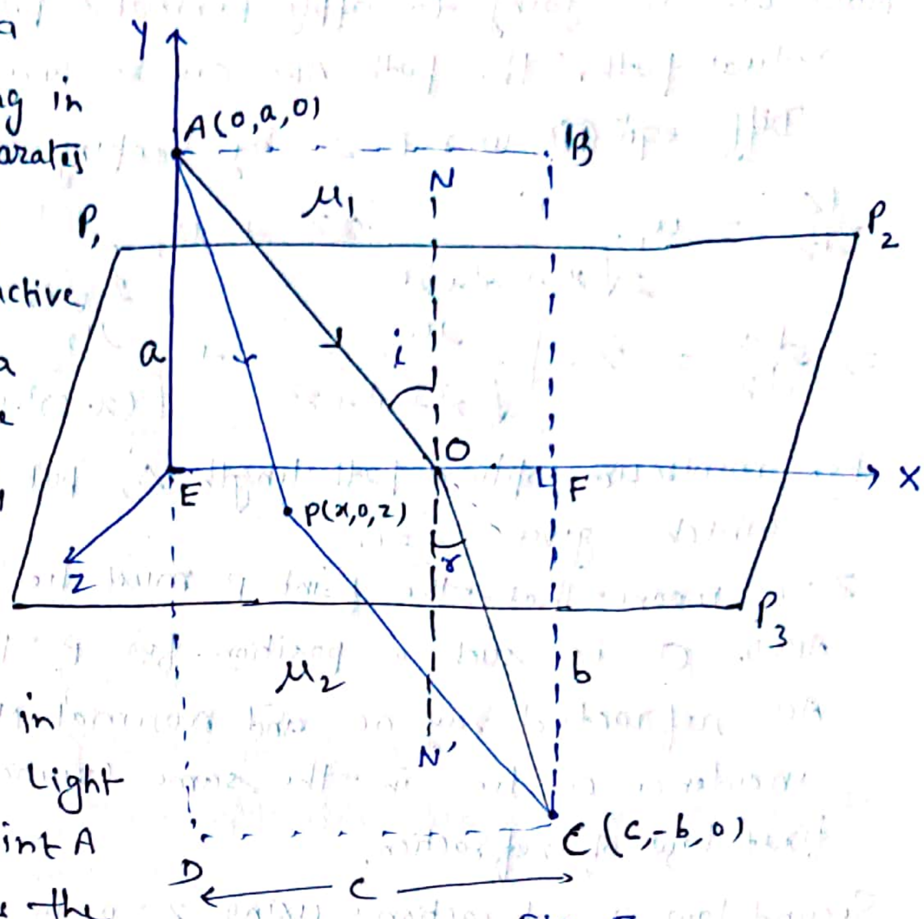

Fig-5

Suppose the light ray passes through a point P in Z-X plane (separating plane). It means that the light ray is incident along AP and it is refracted along PC. Thus APC is most general conceivable path from A to C.

Take E as origin. Let EF and EA are along x and Y axes respectively. EA = a, EF = CD = c, FC = b, and the point P has general coordinates (x, 0, z).

The optical path length through the general point P is given by

$$\Delta = \mu_1 \cdot AP + \mu_2 \cdot PC$$

$$= \mu_1 \cdot \sqrt{(x-0)^2 + (0-a)^2 + (z-0)^2} + \mu_2 \sqrt{(x-c)^2 + (0+b)^2 + (z-0)^2}$$

$$\Delta = \mu_1 \cdot \sqrt{x^2+a^2+z^2} + \mu_2 \cdot \sqrt{(x-c)^2+b^2+z^2} \quad \text{——— (11)}$$

Now we are going to apply Fermat's principle for obtaining actual path. The path APC can be varied by varying x and z.

Diff. equ^n (11) w.r.t. z by keeping x constant

$$\frac{d\Delta}{dz} = \mu_1 \cdot \frac{1}{2\sqrt{x^2+a^2+z^2}} \cdot 2z + \mu_2 \cdot \frac{1}{2\sqrt{(x-c)^2+b^2+z^2}} \cdot 2z$$

$$\Rightarrow \frac{d\Delta}{dz} = z \cdot \left[\frac{\mu_1}{\sqrt{x^2+a^2+z^2}} + \frac{\mu_2}{\sqrt{(x-c)^2+b^2+z^2}}\right] \quad \text{——— (12)}$$

For minimum optical path length Δ, put $\frac{d\Delta}{dz} = 0$ which gives $z = 0$

$z=0$ means that the point P must lie in the vertical plane ABCD. O is such a position for P. Therefore, incident ray AO, refracted ray OC and normal NON' at the point of incidence O lie in the same (vertical) plane ABCD. It is first law of refraction.

Second law of refraction: using $z=0$ in equ^n (11), we get

$$\Delta = \mu_1 \sqrt{x^2+a^2} + \mu_2 \sqrt{(x-c)^2+b^2} \quad \text{——— (13)}$$

Diff. equ^n (13) w.r.t. x, we get

$$\frac{d\Delta}{dx} = \mu_1 \cdot \frac{1}{2\sqrt{x^2+a^2}} \cdot 2x + \mu_2 \cdot \frac{1}{2\sqrt{(x-c)^2+b^2}} \cdot 2(x-c)$$

$$\Rightarrow \frac{d\Delta}{dx} = \mu_1 \cdot \frac{x}{\sqrt{x^2+a^2}} + \mu_2 \cdot \frac{x-c}{\sqrt{(x-c)^2+b^2}} \quad \text{——— (14)}$$

For minimum optical path length, put $\frac{d\Delta}{dx} = 0$

$$\mu_1 \cdot \frac{x}{\sqrt{x^2+a^2}} + \mu_2 \cdot \frac{x-c}{\sqrt{(x-c)^2+b^2}}$$

$$\Rightarrow \mu_1 \cdot \frac{x}{\sqrt{x^2+a^2}} = \mu_2 \cdot \frac{c-x}{\sqrt{(x-c)^2+b^2}} \Rightarrow \mu_1 . \sin i = \mu_2 \cdot \sin r$$

$$\Rightarrow \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = {}_1\mu_2 = \text{constant}$$

$$\left(\because \sin i = \frac{x}{\sqrt{x^2+a^2}}, \quad \sin r = \frac{c-x}{\sqrt{(x-c)^2+b^2}} \right)$$

It is second law of refraction.